## 6.1 Notes: Vectors in a Plane

Quantities such as force and velocity involve both magnitude and direction and cannot be completely characterized by a single real number. To represent such a quantity, you can use a directed line segment. Segment $\overrightarrow{PQ}$ has initial point P and terminal point Q. Its magnitude (or length) is denoted by $\|\overrightarrow{PQ}\|$ and can be found using the Distance Formula. $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Two directed line segments that have the same magnitude and direction are equivalent. The set of all directed line segments that are equivalent to the directed line segment t $\overrightarrow{PQ}$ is a vector v in the plane, written **v** = $\overrightarrow{PQ}$. Vectors are denoted by lowercase, bold face letters such as **u**, **v**, and **w**.

**Ex: 1** Show that **u** and **v** in figure 6.13 are equivalent. (Show both magnitude and direction) usually direction is denoted by an angle

R(1,2) S(4,4) v = $\overrightarrow{RS}$

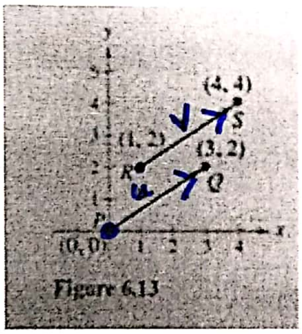


Figure 6.13

$\|\overrightarrow{RS}\| = \sqrt{(4-1)^2+(4-2)^2}$
$\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$ ✓

$\|\overrightarrow{PQ}\| = \sqrt{(3-0)^2+(2-0)^2}$
$\sqrt{3^2+2^2} = \sqrt{9+4} = \sqrt{13}$

$\|v\| = \|u\| = \sqrt{13}$

u = $\overrightarrow{PQ}$

P(0,0) Q(3,2) $m_{RS} = \frac{4-2}{4-1} = \frac{2}{3}$ $m_{PQ} = \frac{2-0}{3-0} = \frac{2}{3}$ ✓

$m_{RS} = m_{PQ}$

**Component Form of a Vector**

The directed line segment whose initial point is the Origin is often the most convenient representative of a set of equivalent directed line segments. This representative of the vector **v** is in standard position.

A vector whose initial point is the origin (0, 0) can be uniquely represented by the coordinates of its terminal point $(v_1, v_2)$. This is the Component form **of a vector v, written as v =** $\langle v_1, v_2 \rangle$. The coordinates $v_1$ and $v_2$ are the components of **v**. If both the initial point and the terminal point lie at the origin, then **v** is the zero vector and is denoted by **0** = $\langle 0, 0 \rangle$.

**Component Form of a Vector**

The component form of the vector with initial point P$(p_1, p_2)$ and terminal point Q$(q_1, q_2)$ is given by

$$\overrightarrow{PQ} = \langle \underbrace{q_1 - p_1}_{\text{horiz. comp.}}, \underbrace{q_2 - p_2}_{\text{vert. comp.}} \rangle = \langle v_1, v_2 \rangle = v$$

The **magnitude** (or length) of **v** is given by

$$\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

If $\|v\|$ = 1, then **v** is a **unit vector.** Moreover $\|v\|$ = 0 if and only if **v** is the zero vector **0.**

**Ex: 2** Find the component form and magnitude of the vector **v** that has the initial point (-3, 2) and terminal point (8, 2).

P (-3, 2)

Q (8, 2)

$v = \langle 8 - {-3}, 2 - 2 \rangle$

$v = \langle 11, 0 \rangle$ ← component form

$\|v\| = \sqrt{(11)^2 + 0^2}$

$\sqrt{121} = 11$

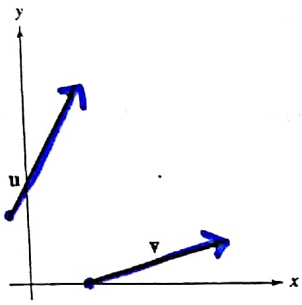
↑ magnitude

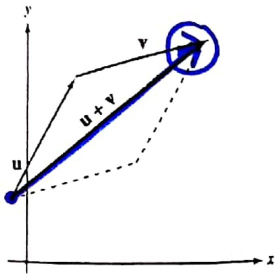

**You Try:** Find the component form and magnitude of the vector **v** that has the initial point (-2, 3) and terminal point (-7, 9).

**Vector Operations**

The two basic vector operations are <u>Scalar multiplication</u> and <u>vector addition</u>. In operations with vectors, numbers are usually referred to as <u>Scalars</u>. In this text scalars will always be real numbers. Geometrically, the product of vector **v** and scalar $k$ is the vector that is $|k|$ times as long as **v**. When $k$ is <u>positive</u>, $k$**v** has the same direction as **v**, and when $k$ is <u>negative</u> $k$**v** has the direction opposite that of **v**. See figure 6.14 in your textbook.

To add two vectors **u** and **v** geometrically, first position them(without changing their lengths or directions) so that the initial point of the second vector **v** coincides with the terminal point of the first vector **u**. The sum **u** + **v** is the vector formed by joining the initial point of the first vector **u** with the terminal point of the second vector **v**.




This technique is called the Parallelogram law for vector addition because the vector **u** + **v**, is often called the resultant of vector addition, is the diagonal of a parallelogram having adjacent sides **u** and **v**.

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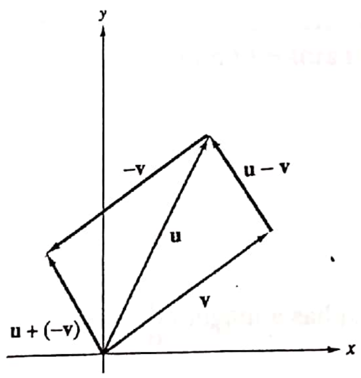
|---|
| **Definitions of Vector Addition and Scalar Multiplication** |
| Let **u** = $\langle u_1, u_2\rangle$ and **v** = $\langle v_1, v_2\rangle$ be vectors and let $k$ be a scalar (a real number). |
| Then the **sum** of **u** and **v** is the vector **u** + **v** = $\langle u_1 + v_1, u_2 + v_2\rangle$ and |
| the **scalar multiple** of $k$ times **u** is the vector $k$**u** = $k\langle u_1, u_2\rangle = \langle ku_1, ku_2\rangle$. |

The **negative** of **v** = $\langle v_1, v_2\rangle$ is $-v = (-1)v = \langle -v_1, -v_2\rangle$ and

The **difference** of **u** and **v** is $u - v = u + (-v) = \langle u_1 - v_1, u_2 - v_2\rangle$.

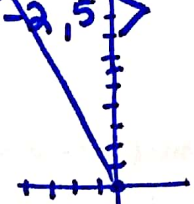



To represent $u - v$ geometrically, you can use directed line segments with the **same** initial point. The difference $u - v$ is the vector from the terminal point of **v** to the terminal point of **u**, which is equal to $u + (-v)$.

**Ex: 3** Let **v** = $\langle -2, 5\rangle$ and **w** = $\langle 3, 4\rangle$. Find each of the following vectors.

a. 2**v** = $2\langle -2, 5\rangle$

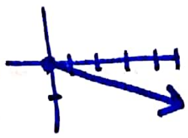
$\langle -4, 10\rangle$



b. **w** − **v**

$\langle 3,4\rangle - \langle -2,5\rangle$

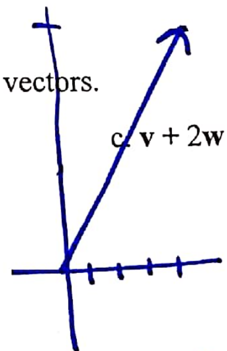
$\langle 5,-1\rangle$



c. **v** + 2**w** = $\langle -2,5\rangle + 2\langle 3,4\rangle$

$\langle -2,5\rangle + \langle 6,8\rangle$

$\langle 4, 13\rangle$



**You Try:** Let **u** = $\langle 1, 4\rangle$ and **v** = $\langle 3, 2\rangle$. Find each of the following vectors. Draw each resultant.

a. **u** + **v**

b. **u** − **v**

c. 2**u** − 3**v**